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‘Serial Concatenated Trellis Coded Modulation with Iterative Decoding: Design and Performance

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Abstract—In this paper, we propose a novel method to design serial concatenation of an outer convolutional code with an inner trellis code with multi-level amplitude/phase modulations and a suitable bit-by-hit iterative decoding structure. Existing two- and multi-dimensional TCM schemes cannot be used directly iu serial concatenated TCM, as they lead to suboptimum pert'ormance. Therefore, a new input labeling method is proposed to construct optimum TCM. Examples are given for a bandwidth efficiency of 2 bits/sec/Hz with $2\times$8PSK oud 16QAM modulations. Inner trellis codes were designed specifically for serially concatenated TCM. A simple iterative decoder for serial concatenated TCM is described. The performance of the code using $2\times$8PSK modulation with a lwo-state inner code, fi-state outer code, and input Mock of 16384 bits is within 0.85 dB from the Shannon limit, at a bit error probability of 10^{-7} for 2 bits/sec/Hz. This low complexity serial TCM outperforms parallel concatenated TCM at low bit error rates for the same complexity. Its performance is similar 10 more complex parallel concatenated TCM employing two 16-slate constituent codes.

1. Introduction

Trellis coded modulation (TCM) proposed by Ungerboeck in 1982 [1] is now a well-estahlishcd technique in digital communications. Since its first appearance, TCM has generated a continuously growing interest, concerning its theoretical foundations as well as its numerous applications, spanning high-rate digital transmission over voice circuits, digital microwave radio relay links, and satellite communications. In essence, it is a technique to obtain significant coding gains (3-6 dB) sacrificing neither data rate nor bandwidth,

Turbo codes represent a more recent development i O the coding research field [2], which has raised a large interest in the coding community. They are *parallel concatenated convolutional codes* (PCCC) whose encoder is formed by two (or more) *constituent* systematic encoders joined through one or more interleavers. The input information bits feed the first encoder and, after having been scrambled by the. interleaver, enter the second encoder. A codeword of a parallel concatenated code consists of the input bits to the first encoder followed by the parity check bits of both encoders.

The suboptimal[1] iterative decoding structure is modular, and consists of a set of concatenated decoding modules, one for each constituent code, connected through the same interleaver used at the encoder side. Each decoder performs weighted soft decoding of the input sequence. Bit error- probabilities as low as 10- “ at $E_b/N_0 = -0.6$ dB have been shown by simulation [4] using codes with rates as low as 1/15. Parallel concatenated convolutional codes yield very large coding gains (10-11 dB) at the expense of a data rate reduction, or bandwidth increase. In [5] we merged TCM and PCCC in order to obtain large coding gains and high bandwidth efficiency. In this paper we suggest to merge TCM with the recently discovered serial concatenated codes (SCCC)[6], adapting the concept of iterative decoding used i n parallel concatenated codes. We note that the proposed serial concatenated coding scheme differs from “classical” concatenated coding systems. In the classical scheme the role of the interleaver between the two encoders is just to separate bursts of errors produced by the inner decoder, and no attempt is made to consider the combination of the two encoders and the interleaver as a single entity. I’bus, the idea behind the recent serial schemes is new since wc want to construct and optimize the whole serial structu t-e, No such attempt was made in the past for two reasons. First, optimizing the overall code with large deterministic interleavers was prohibitively complex. However, by intl-educing the concept of *uniform interleaver [3]* it is possible to draw some criteria to optimize the component codes for the construction of’ powerful serial concatenated codes with large block size. Second, optimum decoding of such complex codes is practically impossible. Only the use of suboptimum iterative decoding methods makes it possible to decode such complex codes. In the following, we will call the concatenation of an outer convolutional code with an inner TCM a *serially concatenated TCM* (SCTCM).

For parallel concatenated trellis coded modulation (PCTCM), also addressed as “turbo ‘I’CM”, a first attempt employing the so-called “pragmatic” approach to TCM was described in [7]. Later, turbo codes were embedded in mult i level codes with multistage decoding [8]. Recentl y, punctured versions of Ungerboeck codes were used to construct turbo codes for 8PSK modulation [9]. In [5] we proposed a ncw solution to PCTCM with multilevel amplitude/phase modulations, and a suitable bit-by-bit iterative decoding structure. Preliminary results [5] showed that the performance of the proposed codes is within I dB from the Shannon limit at bit error probabilities of 10-7.

In [5] we proposed to use for PCTCM two rate $\frac{2b}{2b+1}$ systematic recursive convolutional codes in parallel. The least significant output bit of each encoder represents the parity bit, For each encoder only the $(b+1)$ least significant output bits are mapped to a 2^{b+1}-point constellation (representing the modulation). ‘I’he basic structure of parallel concatenated trellis coded modulation is shown in Fig. I. This method requires at least two interleavers. The first interleaver π_1 permutes the b least significant input bit streams. The output of first interleaver is connected to the b most significant input bits of’ the second encoder. The second interleaver π_2 permutes the b most significant input bit streams. The output of

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[1] Although not fo (tally proved, the suboptimum algorithm yields performance wry close to the maximum-likelihood algorithm [3]

second interleaver is connected to the b least significant input bits of the second encoder.

For MPSK (or MQAM) we use 2^{b+1} PSK symbols (or 2^{b+1} QAM symbols) per encoder to achieve a bandwidth efficiency of b bits/see/Hz. (b bits per modulation symbol). For M-QAM we can also use 2^{b+1} levels in the I-channel and 2^{b+1} levels in the Q-channel, to achieve an efficiency of 2b bits/sec/Hz. The design method we are proposing for SCTCM is different from the one proposed for PCTCM [5].

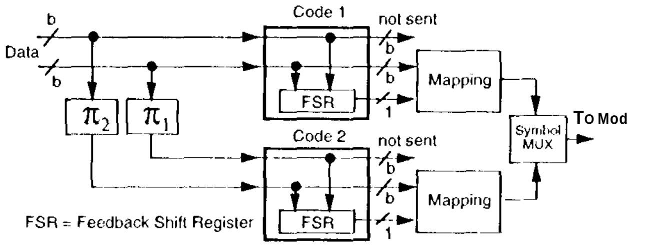

Fig. 1. Block Diagram of the Encoder for Parallel Concatenated Trellis Coded Modulation

II. Serial Concatenated Trellis Coded Modulation

The basic structure of serially concatenated trellis coded modulation is shown in Fig. 2. We propose a novel method to design serial concatenated TCM, which achieves b bits/sec/Hz, using a rate $2b/(2b+1)$ non-recursive binary convolutional encoder with maximum free Hamming distance as the outer code. An interleaver π permutes the output of the outer code. The interleaved data enters a rate $(2b+1)/(2b+2)$ recursive convolutional inner encoder. The $2b+2$ output bits are then mapped to two symbols each belonging to a 2^{b+1} level modulation (four dimensional modulation). In this way, we are using $2b$ information bits for every two modulation symbol intervals, resulting in b bit/sec/Hz transmission (when ideal Nyquist pulse shaping is used) or, in other words, b bits per modulation symbol. The inner code and the mapping will be jointly optimized based on maximizing the effective free Euclidean distance of the inner TCM. in the serial case, however, the inputs to the inner code are not unconstrained bits as in the PCTCM, and thus the design methodology must be different. Since the invention of TCM by Ungerboeck in 1982,

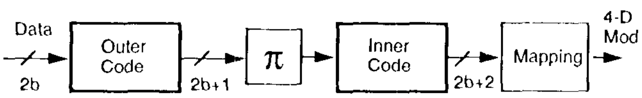

Fig. 2. Block Diagram of the Encoder for Serial Concatenated Trellis Coded Modulation.

there have been numerous papers on the design of two- and multidimensional TCM. Unfortunately, we cannot use the conventional TCM designs for serial or parallel concatenated TCM, even if the structure of the encoder has a feedback as for conventional TCM. There are two main reasons:

1. The first condition to be satisfied for the inner encoder in serial TCM is that the Euclidean distance of encoded sequences be very large for input sequences having Hamming distance equal to 1. This may not be satisfied even if the encoder structure of conventional TCM has feedback. In conventional TCM, in fact, part of the input bits remain uncoded. These bits select a point from a subconstellation (coset) which, in turn, has been chosen according to the encoded bits. The combination of coded and uncoded bits is mapped to two or higher dimensional modulation. One could think of using conventional TCM without parallel branches, but this requires that the number of states be greater than the number of transitions per state, and this, in turn, prevents the use of simple codes with small number of states.
2. For the design of conventional TCM, the assignment of input labels does not play an important role, since it has a small impact on the bit error probability. As a consequence, the input labels assignment was typically arbitrary. For the design of SCTCM (and also for PCTCM), on the opposite, the input label assignment is crucial, as we will see in the following.

A. Design Criteria for Serially Concatenated TCM

It can be shown that the dominant term in the transfer function bound on bit error probability of serially concatenated TCM, employing an outer code with free Hamming distance d_f^o, averaged over all possible interleavers of size N bits, is proportional for large N to

$$N^{-\lfloor (d_f^o+1)/2 \rfloor} e^{-\delta^2 (E_s/4N_o)}$$

where $\lfloor x \rfloor$ represents the integer part of x, and

$$\delta^2 = \frac{d_f^o d_{f,\text{eff}}^2}{2}, \quad \text{for } d_f^o \text{ even}, \quad \text{and}$$

$$\delta^2 = \frac{(d_f^o - 3) d_{f,\text{eff}}^2}{2} + (h_m^{(3)})^2, \quad \text{for } d_f^o \text{ odd}.$$

The parameter $d_{f,\text{eff}}$ is the *effective free Euclidean distance* of the inner code (to be defined in the following). $h_m^{(3)}$ is the minimum Euclidean distance of inner code sequences generated by input sequences with hamming distance 3, and E_s/N_o is the M-ary symbol signal-to-noise ratio.

Previous results were valid for very large N. On the other hand, for large values of the signal-to-noise ratio E_s/N_o, the performance of SCTCM is dominated by

$$N^{-(l_m(h_m)-1)} e^{-h_m^2 (E_s/4N_o)}$$

where h_m is the minimum Euclidean distance of the SCTCM scheme, and $l_m(h_m)$ is the minimum Hamming distance between input sequences to the inner TCM encoder producing h_m. We note that $l_m(h_m) \geq d_f^o$.

Based on the above results, the design criterion for serially concatenated TCM for large interleavers and very low bit error rates is to maximize the free Hamming distance of the outer code, to achieve interleaving gain, and to maximize the effective free Euclidean distance of the inner TCM code.

Let $\mathbf{z}$ be the binary input sequence to the inner TCM code, and $x(\mathbf{z})$ be the corresponding inner TCM encoder output with M-ary symbols. The criteria proposed for designing and selecting the constituent inner TCM encoder are the following:

1. Design the constituent inner TCM encoder for a given two or multidimensional modulation such that the minimum Euclidean distance $d(x(\mathbf{z}), x(\mathbf{z}'))$ over all $\mathbf{z}, \mathbf{z}'$ pairs, $\mathbf{z} \neq \mathbf{z}'$, is maximized, given that the Hamming distance $d_H(\mathbf{z}, \mathbf{z}') = 2$. We call this minimum Euclidean distance the *effective free*

Euclidean distance of the inner TCM code and denote it simply by d_{Leff}.

2. If the free distance of the outer code d_f^o is odd, then, among the selected inner TCM encoders, choose those that have the maximum Euclidean distance $d(x(\mathbf{z}), x(\mathbf{z}'))$ over all $\mathbf{z}, \mathbf{z}'$ pairs, $\mathbf{z} \neq \mathbf{z}'$, given that the Hamming distance $d_H(\mathbf{z}, \mathbf{z}') = 3$. We **call** this the minimum Euclidean distance of the inner TCM code due to input Hamming distance 3, and denote it by $h_m^{(3)}$.
3. Among the candidate encoders, select the one that has the largest minimum Euclidean distance in encoded sequences produced by input sequences with Hamming distance d_f^o. We denote this minimum Euclidean distance of the SCTCM code by h_m.

One may ask why we care about sequences with Hamming distances of 2 or 3 at the input of the TCM encoder if the free Hamming distance d_f^o of the outer code is larger than **2** or even 3. The answer is that the interleaving gain at low SNR depends on the number of error events that a pair 01 input sequences generate in the trellis of the inner code. For a given input Hamming distance, the larger is the number of error events the smaller the interleaving gain wiii bc. g'bus, for example, if d_f^o=4 the largest number of error events is 2 (two events with an input Hamming distance of **2** each).

B. Mapping (output labels) for TCM

As soon as tile input labels and output signals are assigned to the edges of' a trellis we have a complete description of a TCM code. The selection of the mapping (output labels) does not change the trellis code; however, it influences the encoder circu it required to implement the TCM scheme. A convenient mapping should be selected to simplify the encoder circuit and, it possible, to yield a linear circuit that can be implemented with exclusive ORs. The set partitioning of the constellation and the assignment of constellation points to trellis edges, and the successive assignments of input labels to the edges are important. Ungerboeck [i] proposed a mapping called *mapping by set partitioning*, leadi ng to natural mapping. This mapping for two dimensional modulation is useful if one selects the TCM scheme by searching among all encoder circuits that maximize tile minimum Euclidean distance.

C. Design Method for Inner TCM

The proposed design method is based on the following steps:

1. The well known set pat-tit ioning techniques for multidimensional signal sets at-e used (see for example [10] and the references therein).
2. The input labels assignment is based on the codewords of the parity check code ($2b+1$, 2b, 2) and its set partitioning, to maximize the quantities described in subsection A. The assignment of codewords of the parity check code to the 4-dimensional signal points is not arbitrary. We would like somehow to relate the Hamming distance between input labels to the Euclidean distance between corresponding 4-dimensional signal points, under the constraint that the minimum Hamming distance between input labels for parallel transitions be equal to 2. We propose the following method for the input label assignment: Assign the b most significant bits of the codewords to the first constellation with 2^{b+1} points as the b most significant bits of the Gray code mapping; Use the same assignment for the b least significant bits of the code words; The middle bit in the codewords represents the overall parity check bit.
3. A sufficient condition to have very large output Euclidean distances for input sequences with Hamming distance i, is that all input labels to each state be distinct.
4. Assign pair of input labels and 4-dimensional signal points to the edges of a trellis diagram based on the design criteria in subsection A.

To illustrate the design methodology we developed the following examples.

D. Examples of the Design Methodology

Example 1: Set partitioning of **2 x** *8PSK and input labels assignment.* Let the eight phase signals ($e^{j\pi i/4}$; $i = 0, \ldots, 7$) of 8PSK bc denoted by (0, 1,2, 3,4,5,6, 7). Consider the $2\times$8PSK signal set $A_{0,0}$= [(0, 0), (0, **4), (2, 2), (2, 6), (6, 2), (6, 6), (4,** 0), **(4,** 4)] and the following sets constructed from $A_{0,0}$ as: $A_{0,2} = A_{0,0} + (0, 2)$, $B_0 = A_{0,0} \cup A_{0,2}$, $B_1 = B_0 + (0, 1)$, $B_2 = B_0 + (1, 1)$, $B_3 = B_0$ + **(1,** 0), where addition is component-wise modulo 8. For unit radius 8PSK constellation, the minimum intra-set square Euclidean distance of B_i is 2. The minimum inter-set square Euclidean distances are $d^2(B_0, B_2) = d^2(B_1, B_3)$ = 1.17. The other minimum inter-set distances are 0.586. Select the input label set L_0 as codewords of the (5, 4, 2) parity check code. Using tbc method for the input label assignment presented above, we obtain tile assignment of L_0 to B_0. This assignment has been obtained by mapping the first and last **2** bits of input labels to the 8PSK signals as {00, 00, 01, 01, 11, 11, 10, 10} ⇒ {0, 1, 2, 3, 4, 5, 6, 7}. Use the same order and assign L_0 to B_3. The input label set L_1 is assigned to B_1, and B_2, where $L_1 = L_0 + (0, 0, 1, 0, 0)$ modulo 2 according to the above discussed rule. This guarantees that the minimum intra-set Hamming distances of input labels for each set B_i is 2. Assign $(L_0, B_0), (L_1, B_2)$ to the first state, and (L_1, B_1), (L_0, B_3) to the second state (each pair of input label-output signal is assigned to the parallel edges) such that d^2_{Leff} is maximum and, if possible, $h_m^{(3)} = \infty$. The same method is used for the 4-state trellis structure, For 8 and 16-state trellis, wc have to partition B_0 into $A_{0,0}$ and $A_{0,2}$ to obtain larger intra and inter distances, and the input label L_0 is part itioned into L_{00} (codewords of L_0 with b_2=0) and L_{11} (codewords of L_0 with b_2= 1), with similar partitioning for the other input labels and remaining signal sets. Here b_2 represents the middle bit of the (5-bit) input label. Having determined the *trellis code* by its input labels and 4-dimensional output signals, tile encoder structure that generates this trellis code can then be obtained by select i ng any appropriate binary output labels for the 4-dimensional output signals (We used the reordered mapping described in [5]). The truth table required to implement the encoder can be easily obtained, and it is not reproduced here for brevity. From this truth table we obtained the 2-state inner trellis encoder structure shown in Fig. 3(a). For this 2-state inner code, d^2_{Leff} = 1.76, $h_m^{(3)} = \infty$, and h_m^2 = 2. The outer code for the simulations has been selected as an 8-state, rate 4/5, nonrecursive convolutional code with $d_f^o = 3$. Since $h_m^{(3)} = \infty$ then d_f^o is increased effectively to 4.

Example 2: Set *partitioning of 16QAM and input labels assignment.* For this construction wc set b = 1. The number of points in. the 16QAM constellation is $2^{2b/2} = 16$, and thus we obtain an SCTCM with bandwidth efficiency of 2 bits/sec/Hz. Let the amplitudes in each dimension of the 16QAM constellation be denoted by {0, 1, 2, 3}. Consider the signal set $A_{0,0}$ = [(0, 0), (0, 2), (2, 0), (2, 2)] and the following sets constructed from $A_{0,0}$ as: $A_{1,1} = A_{0,0} + (1, 1)$, $A_{0,1} = A_{0,0} + (0, 1)$, $A_{1,0} = A_{0,0} + (1, 0)$. This is precisely the set partitioning adopted by Ungerboeck for 16QAM. For unit power 16QAM constellation, the minimum intra-set square Euclidean distance of each set $A_{i,j}$ is 1.6. The minimum inter-set square Euclidean distances are, instead, $d^2(A_{0,0}, A_{1,1}) = d^2(A_{0,1}, A_{1,0}) = 0.8$. The other minimum interdistances are 0.4. Select the input label set L_0 as codewords of the (3, 2, 2) parity check code. Assign these codewords to the sets $A_{0,0}$, and $A_{1,0}$. This was done based on the input labeling

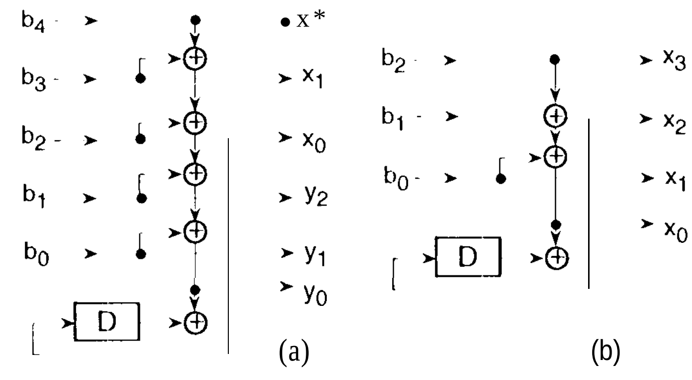

Fig. 3. Optimum 2-state inner trellis encoder for SCTCM (a) 2×8PSK, (b) 16QAM

method described above, by mapping the MSB and the LSB of the 3-bit input labels to the 4 amplitude levels per dimension of the 16QAM points as {0, 0, 1, 1} ⇒ {0, 1, 2, 3}. The input label set L_1 is assigned to $A_{1,1}$, and $A_{0,1}$, where $L_1 = L_0 + (0, 1, 0)$ modulo 2. This guarantees that the minimum Hamming distances of input labels is 2 for each set $A_{i,j}$. Assign $(L_0, A_{0,0}), (L_1, A_{1,1})$, to the first state, and $(L_1, A_{0,1}), (L_0, A_{1,0})$ to the second state, such that $d^2_{f,eff}$ is maximum, and, if possible, $h_m^{(3)} = \infty$. A similar design method was used for the 4-state trellis structure. For 8 and 16-state trellises, we have to partition $A_{0,0}$ into [(0, 0), (2, 2)] and $A_{1,1}$ into [(0, 2), (2, 0)] to obtain larger intra and inter-set distances, and the input label L_0 into L_{00} (codewords of L_0 with $b_1 = 0$) and L_{11} (codewords of L_0 with $b_1 = 1$), with a similar partitioning for the other input labels and remaining signal sets. Here too b_1 is the middle bit of the input label. Having determined the code by its input labels and 2-dimensional output signals, the encoder structure is then defined by selecting any appropriate binary output labels for the 2-dimensional output signals (we used the natural mapping per dimension of 16QAM). The implementation of the 2-state inner trellis encoder is shown in Fig. 3(b). For this 2-state inner code, $d^2_{f,eff} = 1.2$ and $h_m^{(3)} = \infty$. For the simulations, a 16-state, rate 2/3, nonrecursive convolutional code with $d_f^o = 5$ has been selected as an outer code.

III. Bit-by-Bit Iterative Decoding of Serially Concatenated Trellis Coded Modulation

The iterative decoder for serially concatenated trellis coded modulation uses a generalized Log-APP (a-posteriori probability) decoder module with four ports, called SISO APP module or simply SISO [11]. The block diagram of the iterative decoder for serial concatenated TCM is shown in Fig. 4. We briefly describe the

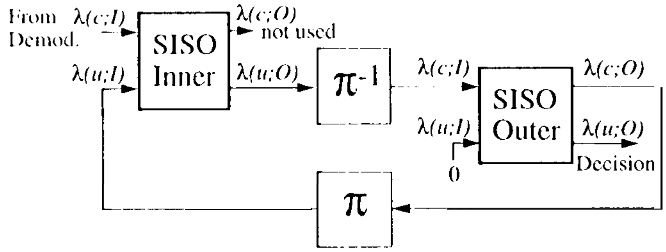

Fig 4. Iterative decoder for serially concatenated trellis coded modulation.

SISO algorithm for the inner TCM code and outer convolutional code, using the trellis section shown in Fig 5. Consider an inner TCM code with p_1 input bits and q_1 nonbinary complex output symbols with normalized unit power, and an outer code with p_2 input bits and q_2 binary outputs {0, 1}. Let $\mathbf{u}_k(e)$ represent $u_{k,i}(e)$; $i = 1, 2, \ldots, p_m$), the input bits on a trellis edge at time k ($m = 1$ for the inner TCM, and $m = 2$ for the outer code), and let $\mathbf{c}_k(e)$ represent $c_{k,i}(e)$; $i = 1, 2, \ldots, q_m$ the output symbols ($m = 1$ for the inner TCM, with nonbinary complex symbols, and $m = 2$ for the outer code with binary {0, 1} symbols).

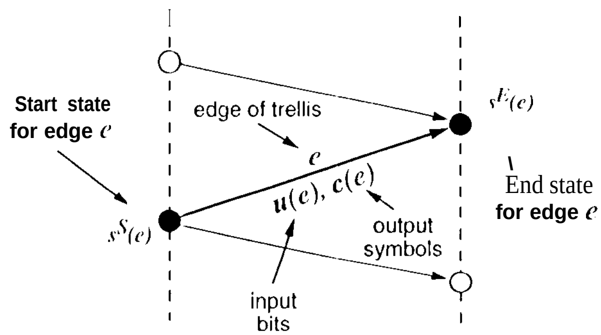

Fig. S. Trellis Section

Define the reliability of a bit Z taking values {0, 1} at time k as

$$\lambda_k[Z; \cdot] \triangleq \log \frac{P_k[Z = 1; \cdot]}{P_k[Z = 0; \cdot]}$$

The second argument in the brackets, shown as a dot, may represent I, the input, or O, the output, to the SISO. We use the following identity

$$a = \log[\sum_{i=1}^{L} e^{a_i}] = \max_i\{a_i\} + \delta(a_1, \ldots, a_L) \triangleq \max_i{}^*\{a_i\}$$

where $\delta(a_1, \ldots, a_L)$ is the correction term which can be computed using a look-up table. For more detail see [12] and its references number 25, 26, 31, and the reference in footnote 1. We define the "max*" operation as a maximization (compare/select) plus a correction term (lookup table). Small degradations occur if the "max*" operation is replaced by "max". The received complex samples $\{y_{k,i}\}$ at the output of the receiver matched filter are normalized such that additive complex noise samples have unit variance per dimension.

A.. The S1S0 Algorithm for the Inner TCM

The *forward* and the *backward* recursions are:

$$\alpha_k(s) = \max^*_{e:s^E(e)=s} \{\alpha_{k-1}[s^S(e)] + \sum_{i=1}^{p_1} u_{k,i}(e)\lambda_k[U_{k,i}; I] + \sum_{i=1}^{q_1} \tilde{\lambda}_k[c_{k,i}(e); I]\} + h_{\alpha_k}$$

$$\beta_k(s) = \max^*_{e:s^S(e)=s} \{\beta_{k+1}[s^E(e)] + \sum_{i=1}^{p_1} u_{k+1,i}(e)\lambda_{k+1}[U_{k+1,i}; I] + \sum_{i=1}^{q_1} \tilde{\lambda}_{k+1}[c_{k+1,i}(e); I]\} + h_{\beta_k}$$

for all states s, anti $k =$ i, ... $(n-1)$, where n represents the total number of trellis steps from the initial state to the final state. The *extrinsic bit information* for $U_{k,j}$; j = 1, 2. ... p , can be obtained from:

$$\lambda_k(U_{k,j}; O) = \max^*_{e:u_{k,j}(e)=1} \{\alpha_{k-1}[s^S(e)] + \sum_{i=1, i\neq j}^{p_1} u_{k,i}(e)\lambda_k[U_{k,i}; I] + \sum_{i=1}^{q_1} \tilde{\lambda}_k[c_{k,i}(e); I] + \beta_k[s^E(e)]\}$$
$$- \max^*_{e:u_{k,j}(e)=0} \{\alpha_{k-1}[s^S(e)] + \sum_{i=1, i\neq j}^{p_1} u_{k,i}(e)\lambda_k[U_{k,i}; I] + \sum_{i=1}^{q_1} \tilde{\lambda}_k[c_{k,i}(e); I] + \beta_k[s^E(e)]\}$$

where $\tilde{\lambda}_k[c_{k,i}(e); I] = -|y_{k,i} - \sqrt{E_s}\, c_{k,i}(e)|^2/2$. We assume the initial and the final states of the inner encoder (as well as the outer encoder) are the all zero state. Forward recursions start with initial values, $\alpha_0(s) = 0$, ifs = O (initial zero state) and $\alpha_0(s) = -\infty$, if s # 0. Backward recursions start with $\beta_n(s) = 0$, if $s = 0$ (final zero state) anti $\beta_n(s)$ = --co, if s # 0. The h_{α_k} and h_{β_k} are normalization constants which, in the hardware implementation of tbe S1S0, are used to prevent buffer overflow. These operations are similar to the Viterbi algorithm used in the forward and backward directions, except for a correction term that is added when compare-select operations are performed. At the first iteration ail $\lambda_k[U_{k,i}; I]$ are zero. After the first iteration, the inner S1S0 accepts the extrinsics from the outer S1S0, through the interleaver π, as reliabilities of input bits of TCM encoder, and the external observations from the channel. The inner S1S() uses the input reliabilities and observations for the calculation of new extrinsics $\lambda_k(U_{k,j}; O)$ for the input bits. These are then provided to the outer SISO module, through the deinterleaver π^{-1}.

B. The S1S0 Algorithm *for* the Outer Code

The *forward* and the *backward* recursions are:

$$\alpha_k(s) = \max^*_{e:s^E(e)=s} \{\alpha_{k-1}[s^S(e)] + \sum_{i=1}^{q_2} c_{k,i}(e)\lambda_k[C_{k,i}; I]\} + h_{\alpha_k}$$

$$\beta_k(s) = \max^*_{e:s^S(e)=s} \{\beta_{k+1}[s^E(e)] + \sum_{i=1}^{q_2} c_{k+1,i}(e)\lambda_k[C_{k+1,i}; I]\} + h_{\beta_k}$$

The *extrinsic information* for $C_{k,j}$; $j = 1, 2, \cdots, q_2$, can be obtained from:

$$\lambda_k(C_{k,j}; O) = \max^*_{e:c_{k,j}(e)=1} \{\alpha_{k-1}[s^S(e)] + \sum_{i=1, i\neq j}^{q_2} c_{k,i}(e)\lambda_k[C_{k,i}; I] + \beta_k[s^E(e)]\}$$
$$- \max^*_{e:c_{k,j}(e)=0} \{\alpha_{k-1}[s^S(e)] + \sum_{i=1, i\neq j}^{q_2} c_{k,i}(e)\lambda_k[C_{k,i}; I] + \beta_k[s^E(e)]\}$$

with initial values, $\alpha_0(s) = 0$, if $s = 0$ anti $\alpha_0(s)$ = –cm, if s # 0, and $\beta_n(s) = 0$, if s = 0 and $\beta_n(s) = -\infty$, if s # 0, where h_{α_k} and h_{β_k} arc normalization constants which, in the hardware implementation of the S1S0, arc used to prevent the buffer overflow.

The final decision is obtained from the bit reliability computation of $U_{k,j}$; $j = 1, 2, \cdots, p_2$, passing it through a hard limiter, as

$$\lambda_k(U_{k,j}; O) = \max^*_{e:u_{k,j}(e)=1} \{\alpha_{k-1}[s^S(e)] + \sum_{i=1}^{q_2} c_{k,i}(e)\lambda_k[C_{k,i}; I] + \beta_k[s^E(e)]\}$$
$$- \max^*_{e:u_{k,j}(e)=0} \{\alpha_{k-1}[s^S(e)] + \sum_{i=1}^{q_2} c_{k,i}(e)\lambda_k[C_{k,i}; I] + \beta_k[s^E(e)]\}$$

The outer S1S0 accepts the extrinsic from the inner S1S0 as input reliabilities of coded bits of the outer encoder. For the outer S1S[) there is no external observation from the channel. The outer S1S0 uses the input reliabilities for calculation of new extrinsic $\lambda_k(C_{k,j}; O)$ for coded bits. These arc then provided to the inner SISO module.

IV. Simulation of Serial Concatenated Trellis Coded Modulation with Iterative Decoding

in this section the simulation results for serial concatenated TCM, with 2 x 8PSK and 16QAM are presented. For SCTCM with 2 x 8PSK, the outer code is a rate 4/5, fl-state nonrecursive convolutional encoder with d_f^o = 3,andtiu3inncr codcistile 2-state TCM designed for 2 x 8PSK in section II. The bit error probability vs. number of iterations for' various bit signal-to-noise ratios is shown in Fig. 6. The hit error probability vs. bit signal-to-noise ratio E_b/N_o for various numbers of iterations is shown in Fig. 7. This example demonstrates the power and bandwidth efficiency of SCTCM, in particular at low bit error rates,

For comparison with parallel concatenated TCM, we consider the performance of the 2 bits/sec/Hz PCTCM with 8PSK and reordered mapping proposed in [5]. The code has b = 2, and employs 8PSK modulation in connection with two 16-state, rate 4/5 constituent codes. Two random interleavers, each of size 8192 bits were used (total input block of 16384), The bit error probability performance of the selected code is shown in Fig. 8.

SCTCM simulations arc for 16QAM at 2 bits/sec/Hz, where the outer code is a rate 2/3, 16-state nonrecursive convolutional code with $d_f^o = 5$, and the inner code is the 2-state TCM designed for 16QAM in section II. The bit error probability vs. number of iterations for various bit signai-to-noise ratios is shown in Fig. 9.

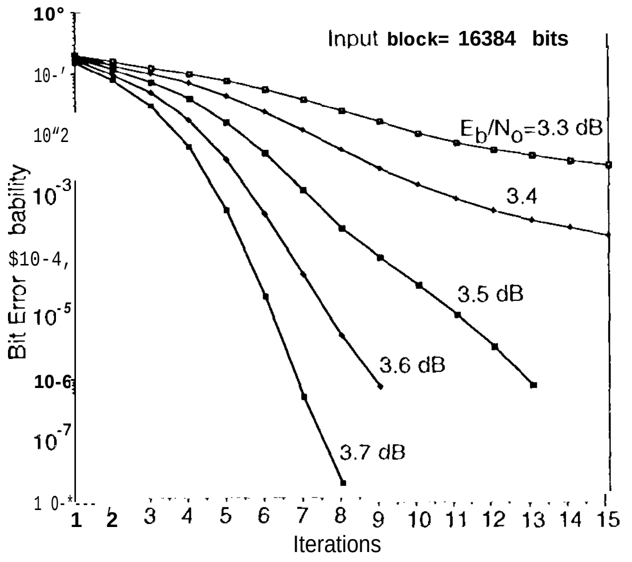

Fig. 6. Performance of Serial Concatenated Trellis Coded Modulation, 8-state outer 2-state inner with 2 x 8PSK, 2 bits/sec/Hz: BER vs number of iterations

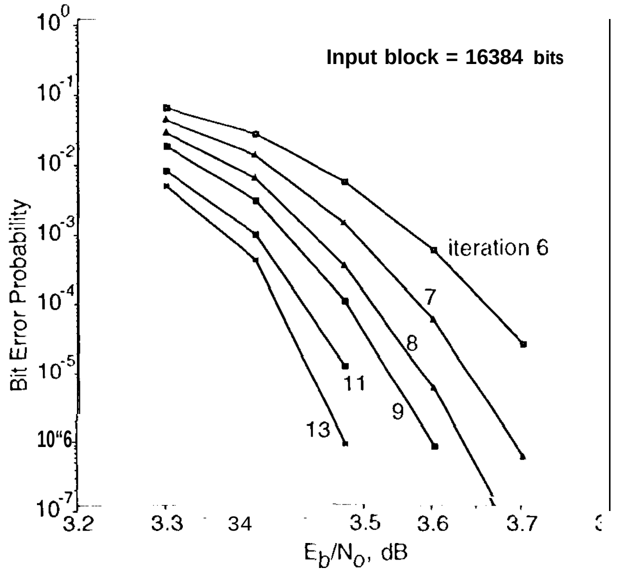

Fig. 7. Performance of Serial Concatenated Trellis Coded Modulation, 8-state outer 2-state inner with 2×8PSK, 2 bits/sec/Hz: BER vs E_b/N_o

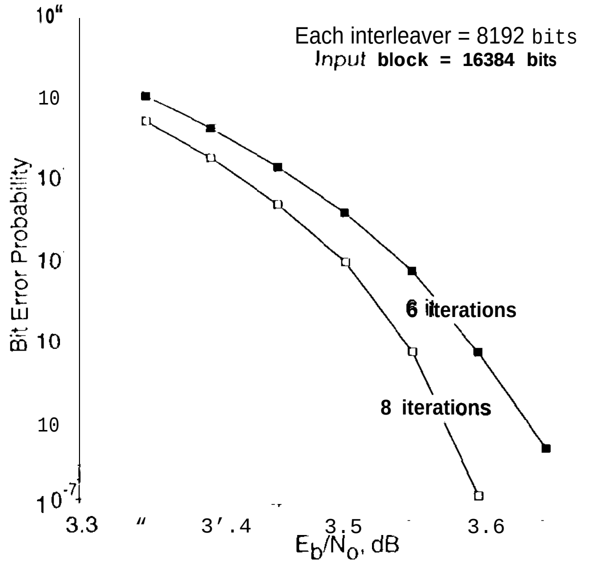

Fig. 8. BER Performance of Parallel Concatenated Trellis Coded Modulation, 8PSK, 2 bps/Hz.

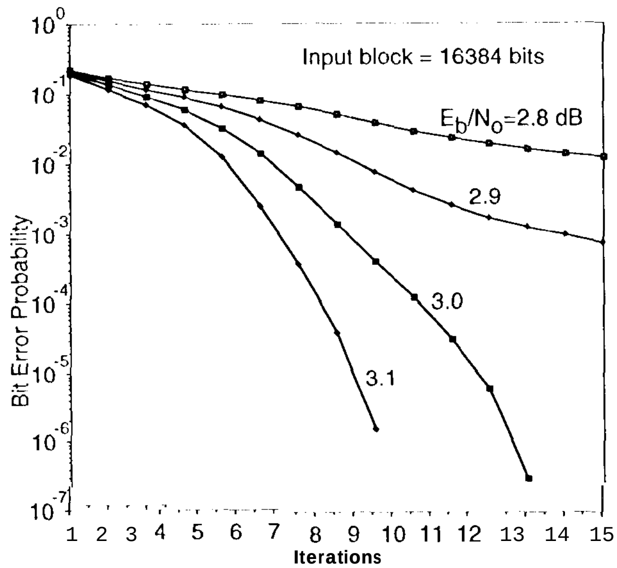

Fig. 9. Performance of Serial Concatenated Trellis Coded Modulation, 16-state outer, 2-state inner, with 16 AM, 2 bits/sec/Hz BER vs. number of iterations

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